Why know anything about linear algebra?

Let $f: \mathbb{R} \to \mathbb{R}$ be a differentiable function. Then "near" the point a, f is "almost" equal to its tangent line

$$y = f'(a)(x-a) + f(a)$$

So $f(x) - f(a)$ is "almost"

$f'(a)(x-a)$

$\boxed{f'(a) \text{ is a } 1 \text{ by } 1 \text{ matrix}}$

Similarly, if $f: \mathbb{R}^n \to \mathbb{R}^m$ is differentiable, the **total derivative** of f at a point $a \in \mathbb{R}^n$ is an $m \times n$ matrix

Df_a.

Concrete Applications

1) Ohm's Law, Kirchoff Current and Voltage Laws

V = voltage

I = current

R = resistance

Ohm's Law: $I = \frac{V}{R}$

Kirchoff's Current Law

If I_k is the current flowing into node k and if there are n nodes in the circuit, then

$$\sum_{k=1}^{n} I_k = 0$$

Kirchoff's Voltage Law

If V_k is the electric potential difference between two nodes and there are m connections in the circuit,

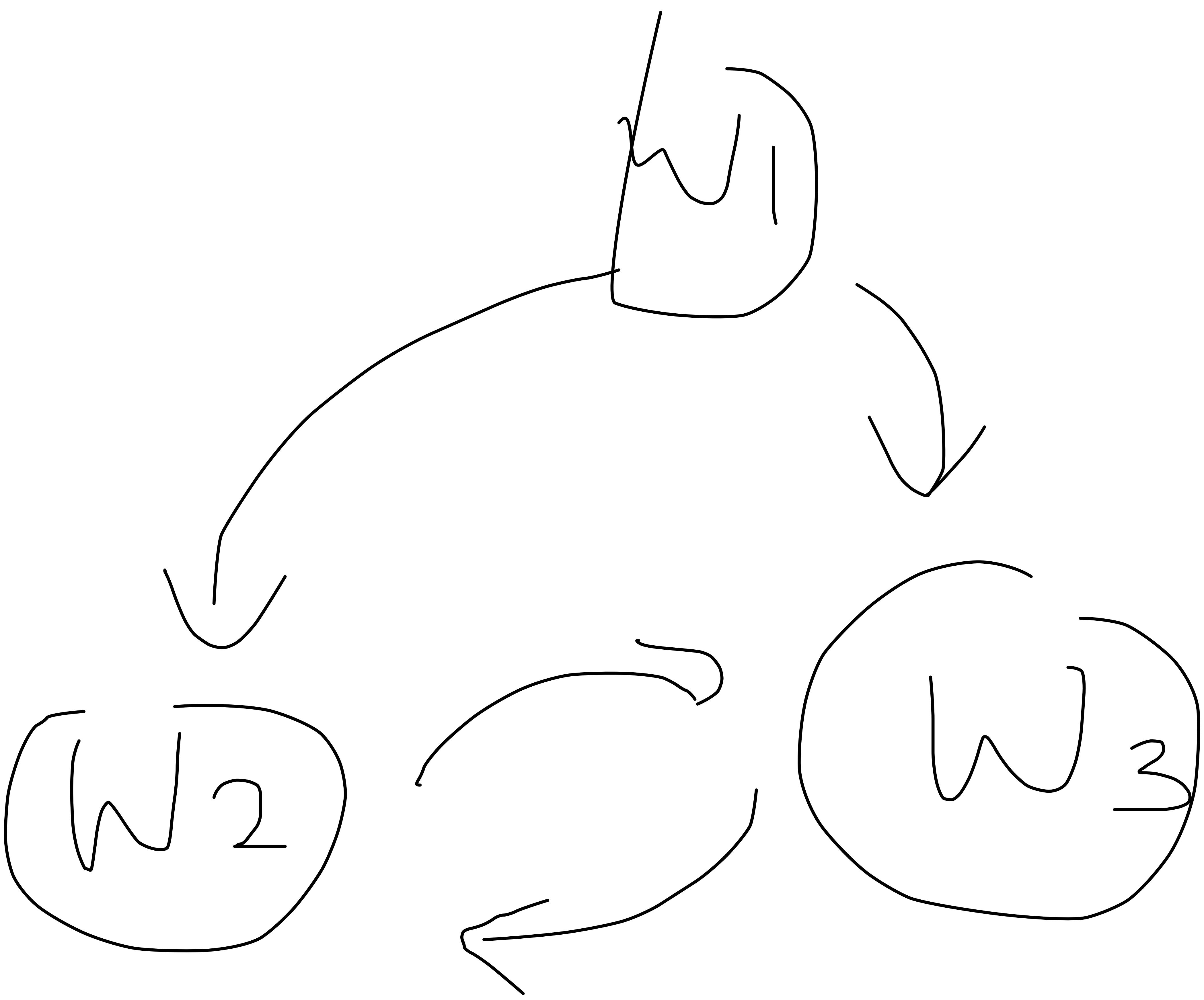
$$\sum_{k=1}^{m} V_k = 0$$

We will do all this using matrices

2) Google's PageRank

Given webpages $W_1, W_2, \ldots, W_n$, construct an n by n square matrix with all nonnegative entries, (i,j) entry is equal to one if there is a link from W_j to W_i, zero if there is no link

Picture



Matrix is 3×3

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Perron-Frobenius Theory gives you a vector $v = (v_1, \ldots, v_n)$ such that each v_i is nonnegative and represents the probability that a "random surfer" will arrive at the page W_i.

This is Google's PageRank algorithm in essence

3) Fast Fourier Transform

Given a signal (radio, phone, etc) that is band-limited

- frequency components are bounded.

We can think of the signal as a continuous function f on $(-\infty, \infty)$. Suppose $\int_{-\infty}^{\infty} |f(x)|\,dx$ is finite

Nyquist's lemma gives us that f is determined by its values at finitely many points.

We then reconstruct f via the discrete Fourier transform.

With 2 points, the discrete Fourier transform is the matrix

$$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

(sometimes scaled by $\frac{1}{\sqrt{2}}$)

For n points, the Fourier transform is an n by n matrix,

4) Compressed Sensing

Given a system of linear equations with more unknowns than equations, if there is a unique "sparse" (= many zero entries) solution, then the solution may be recovered

Image recognition, resolution (lo to hi)

Fourier transform again comes into play.

5) Special Relativity

Space-time position given by a 4-vector

$v = (v_1, v_2, v_3, v_4)$

Minkowski length of v

$$\left| -v_1^2 + v_2^2 + v_3^2 + v_4^2 \right|^{1/2}$$

Lorentz transformations are (some) of the length preserving transformations

These are matrices

6) Quantum Mechanics

Measurement and time-evolution operations are given by matrices – but they may have infinitely many entries!